Proportionality and Unit Rates

Mathematical Focus
- (6.RP.3) Use ratio and rate reasoning to solve real-world and mathematical problems.
- (7.RP.2b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Potential Challenges and Misconceptions
The ability to determine and compare unit pricing is a critical consumer skill, yet so often consumers are confused by such information, particularly when measurement units are involved. Often people assume that the larger a container of food is, the better the price. Yet if they checked the unit rate for different-sized containers, they could make a more informed decision and recognize that it may be less expensive to buy two smaller containers of a particular product rather than one larger container. Multiple representations can help students better understand the proportional thinking associated with unit rates. Students may also overgeneralize rules they have learned about rounding. Make sure students understand that in a store, money is always rounded up to the nearest cent.

In the Classroom
Begin by asking students how much they would pay for one pencil if they bought pencils at a sale price of two for $0.99. Make sure students agree that one pencil would cost $0.50, as stores round up prices to the closet whole cent. Then present the following problem to students:

> Apple Snacks are on sale today. You can buy 4 Apple Snacks for $5. Choose four other numbers of Apple Snacks you might buy. What would be their total cost?

Divide the class into three groups. Have the students in one group represent this information in tables, students in the second group use tape diagrams, and students in the third group use graphs. The students in each group can work alone or in pairs. Ask one student or pair from each group to present its work to the class. Ask questions like: *Where is the one piece of information you were given? How did you use this information to include other equivalent ratios? Which ratio shows the unit price?* (Or, if the unit price is not included in the students' work, *How could you use this information to find the unit price?*) *Are all of the ratios shown equivalent? How do you know?*

To emphasize to students that when all of the ratios are equivalent, the relationship is proportional, add a counterexample, such as twelve Apple Snacks for sixteen dollars, and ask whether it fits their data.

Present students the following problems, or use real local grocery flyers and create your own, similar questions:

3 cans of corn for $3.50	Turkeys: 12 lb. for $8.99	Bananas: $0.69 per lb.
What does 1 can cost?	What does 1 lb. cost?	What do $7\frac{1}{2}$ lb. cost?

Circulate as student pairs are working, posing questions such as *Can you find another way to solve that? Can you convince me? How does the turkey problem relate to the banana question? How do you know your solution is correct? Why are you rounding up?*

After an allotted period of time, invite student volunteers to share the strategies and representations they used, their solutions, and how the three problems are similar and dissimilar. When discussing the solutions to the problems, be sure that the students understand that the first two problems are asking the same thing—to find the unit rate—and that the third question *gives the unit rate* and asks students to use that to find the *total cost*.

Provoke a conversation that prods students to articulate why they need to divide for the first two problems but have to multiply for the third. Ask how the division and multiplication connect to their representations. Ask them how they might convince someone who disagrees with their solutions or choice of operations. Finally, make sure students understand that unit rates can be used to compare ratios. After this discussion, assign the *Camping Out* reproducible on page A24 of the appendix.

Meeting Individual Needs
For students who are uncertain about how to use each of the representations (table, tape diagram, and graph) for these problems, provide them with a copy of the sample solutions in the *Possible Representations* reproducible on page A25. The reproducible illustrates solutions for the three grocery-ad problems presented earlier. Tell the students to use those samples as a reference sheet as they work toward becoming more proficient with these models.

REFERENCE/FURTHER READING
Telese, James A., and Jesse Abete Jr. 2002. "Diet, Ratios, Proportions: A Healthy Mix." *Mathematics Teaching in the Middle School* 8 (1): 8–13.

Name: Date:

In preparing for a weekend camping trip, you and your best friend have to go shopping for ground beef, corn, and hot dogs. Since you do not have a lot of money, you compare the advertisements from two grocery stores, Food for Less and Grocery Bargains.

Examine the following ads and determine which is the cheaper option for each food or if the options are equivalent. Show all your work and justify your answers.

Food for Less		*Grocery Bargains*
Hamburger Patties		Hamburger Patties
5lb. for $8.99		7lb for $10.99

Food for Less		*Grocery Bargains*
Corn		Corn
8 ears for $11		12 ears for $15

Food for Less		*Grocery Bargains*
Hot Dogs		Hot Dogs
1lb. for $4.99, 1lb. free		1lb. for $3.99

Name: Date:

Add two representations.

Table

Cans	Cost in $
1	1.17
3	**3.50**
6	7.00
9	10.50

Diagram

3 cans of corn at $3.50
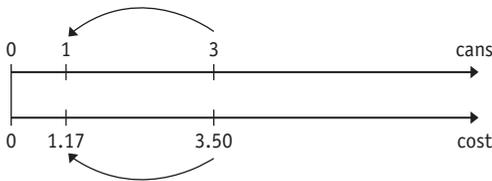
or $3.50 ÷ 3 = $1.17 per can

Graph
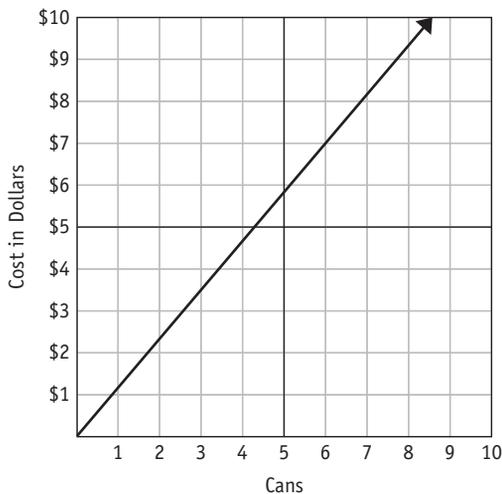
Table

Pounds	Cost in Dollars
1	0.75
3	2.75
6	5.50
12	**8.99 or 9.00**

Diagram

12 lb. of turkey at $8.99 (round up to $9.00)
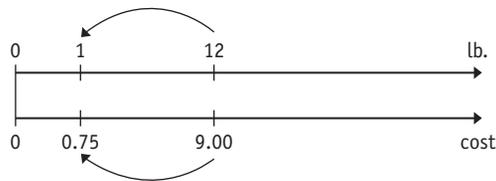
or $9.00 ÷ 12 lb. = $0.75 per lb.

$7\frac{1}{2}$ lb. bananas at $0.69 per lb.

$0.69 × $7\frac{1}{2}$ lb. = $5.18

Graph

A25