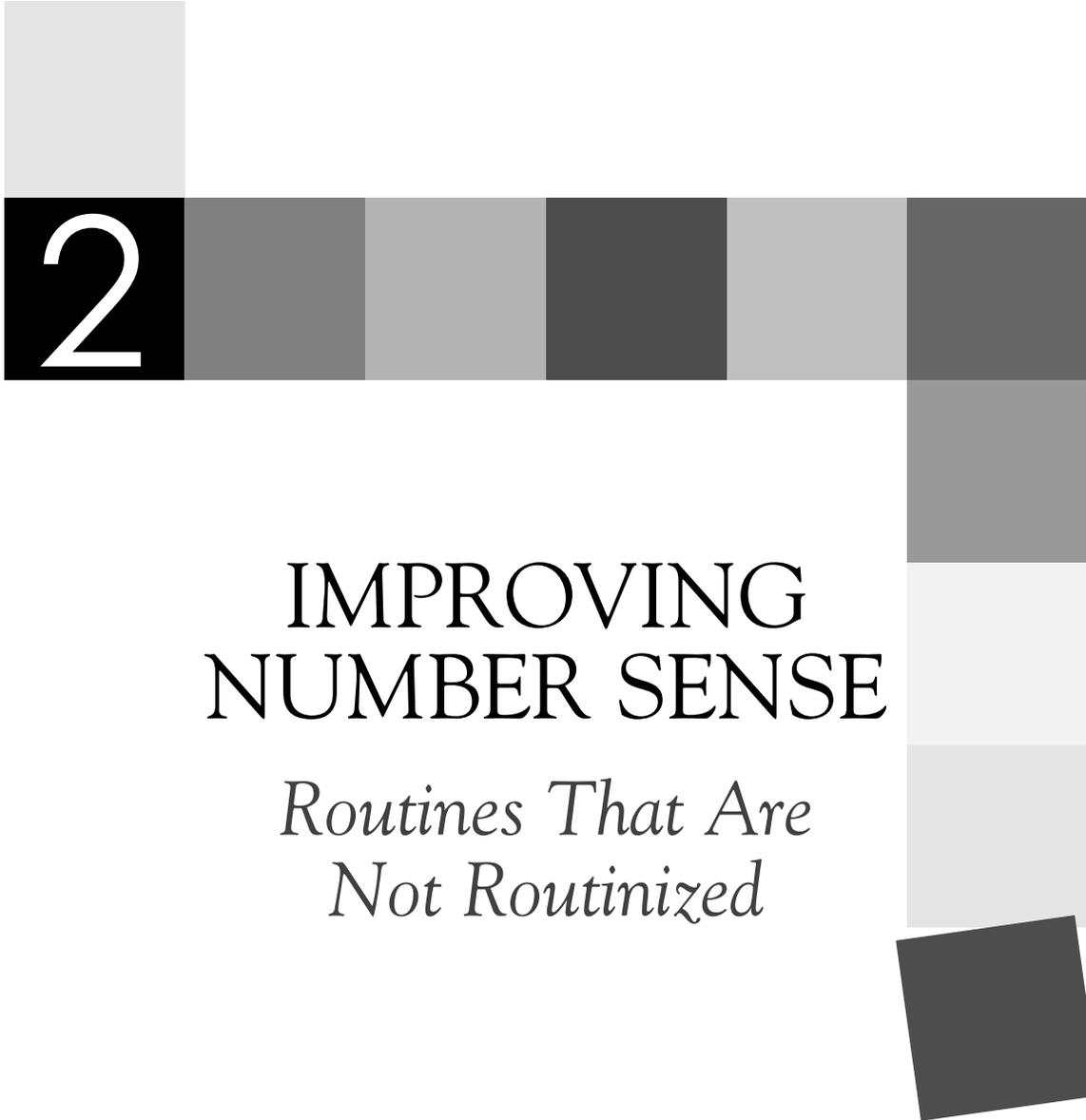
A decorative background consisting of a grid of colored squares in various colors including red, orange, yellow, green, blue, purple, black, and white, arranged in a pattern that partially obscures the text.

Jessica F. Shumway

NUMBER SENSE ROUTINES

Building Numerical
Literacy Every Day
in Grades **K-3**

Foreword by Lucy West



2

IMPROVING NUMBER SENSE

Routines That Are Not Routinized

A routine is an activity or event that occurs on a regular basis over a period of time. Think about the routines you already have in your classroom—greeting students in the morning, backpack procedures, morning meeting rituals, taking attendance, lunch count and lunch line procedures, read-alouds, calendar routines, weather observations and graphs, and author’s share, just to name a few. Routines provide frameworks for our day. Our routines build community and create a safe learning environment for students. Routines provide feelings of belonging, ownership, and predictability, which make the classroom a place to take risks, try new things, and be successful.

Routines are a regular part of most math workshops and math lessons. You find them in curriculum materials, such as the math message in *Everyday Mathematics* (University of Chicago School Mathematics Project 2007) and ten-minute math in *Investigations in Number, Data, and Space* (TERC 2008). Many teachers begin their math block with some kind of warm-up. My purpose is to help you take what you are already doing with math routines and refine it to expand students’ number sense. In this book I show you how to go beyond the curriculum materials to design routines

based on your students' unique strengths and needs. These number sense routines are not “auto pilot” activities, but opportunities for meaningful practice. You'll learn when to use a particular routine, how to differentiate, and how to use routines as formative assessment tools. We'll also explore the mathematics behind the routines and take a look at paths students take as they develop their number sense.

A ROUTINE IN ACTION: COUNT AROUND THE CIRCLE

I shook the rain stick, our signal to clean up from quiet time and transition into math workshop. My fourth-grade students put their materials away and made their way over to the community circle. As Jose plopped down beside me, he asked, “Are we counting by hundreds today?” I gave him the heads up that we would be counting backward by tens. He began counting backward from ninety by tens quietly to himself as his classmates got settled.

Within two minutes everyone was ready. They were sitting in a circle on the floor and were ready to “count around the circle.” I began our routine by saying, “Let's start with 188 and count backward by tens around the circle. If I start with 188 and we move clockwise around the circle, what do you think Catie will land on?” Catie was sitting directly across from me, about halfway around our circle of twenty-two students.

Anthony estimated, “Somewhere in the hundreds, like close to 118 or 108, because we'll go pretty far down the number line if we are counting by tens.”

Marjorie said, “Maybe close to Anthony's guess, but maybe a little less than 100.”

Nisaa added on to Marjorie's idea and said, “I agree with Marjorie, because Catie's about the tenth person and we're counting by tens. That means that it will be about 100 less than 188 . . . so, around 88?”

We had a quick discussion reinforcing the idea that an estimate does not have to be exact by looking at words that Anthony, Marjorie, and Nisaa used: *somewhere*, *close*, *about*, *around*, and *maybe*. We briefly talked about why numbers like 178 and 268 would not be good estimates. The number 178 is only one jump of ten away from 188—that estimate did not make sense because the first person to count would say that number. The number 268 is more than 188—this wouldn't make sense because we said we would be counting backward, not forward.

I started the count for that day's sequence by saying, “Let's try it . . . 188.”

Jose, the first person, said, “One hundred seventy-eight,” and then we continued around the circle. I wrote each number on an open number line as someone said it (see Figure 2.1). The visual scaffold was helpful for the majority of the class, although in different ways. It helped the few students who were still struggling with this skill of counting backward by tens, as it allowed them

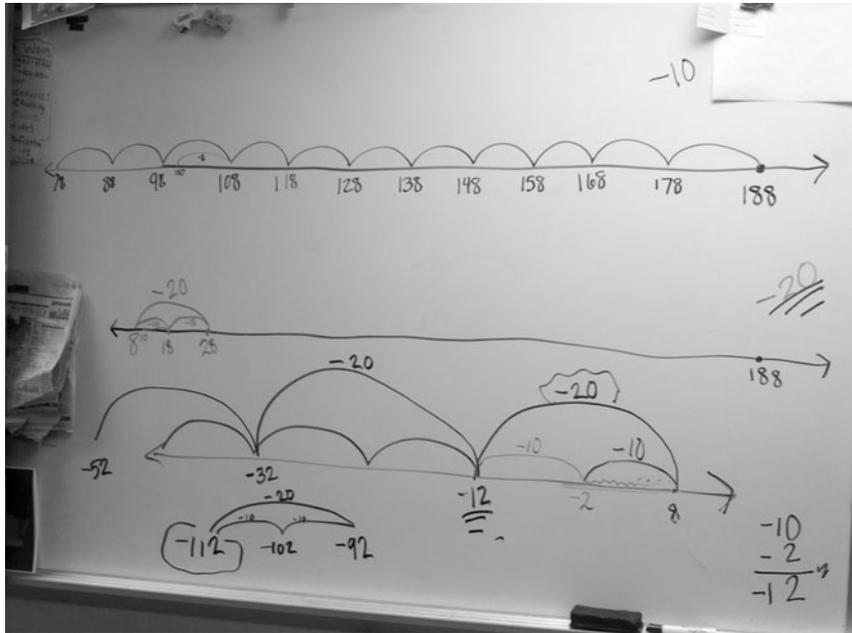


Figure 2.1

to participate in the counting activity. The majority of the students, however, did not need the visual scaffold to support their skill of counting backward by tens. But it helped these students to really understand the pattern and later apply it to other situations, for example, counting backward by twenty.

We continued to count smoothly around the circle, each person saying a number aloud while everyone else counted in their heads: “One hundred sixty-eight, 158, 148 . . .” Then, Adam got stuck. Adib, the person before him, said, “One hundred eight.” Everyone waited silently, knowing that Adam would figure it out. He looked at Anthony and restated “One hundred twenty-eight,” then looked at Melanie and restated “118,” then restated Adib’s number, “108.” He said, “One hundred?” I wrote 100 on the number line, showing that it was 8 away from 108. That visual scaffold on the open number line was just enough support, and he said, “Ninety-eight!” Adam was one of the students still working to keep the visual number line model in his head. He wasn’t quite fluent and automatic yet; nevertheless, he was able to solve the problem.

When we got to Catie, she said, “Seventy-eight,” and we all nodded, confirming our estimates. I stopped them when we got to 8 in order to revisit the estimates and talk about Adam’s strategy for figuring out the jump to 98.

Then, we tried counting backward by twenty, this time all the way around the circle. I again drew the open number line, but did not write each number as the students counted. I encouraged them to “see the jumps” in their heads as we counted around the circle. When Antonio got stuck, I drew a support on the number line to scaffold his strategy for figuring out what would come next (see Figure 2.1). We held a brief discussion about how the pattern changed after zero (8, -12, -32, . . .), how the tens place was no

longer even but now odd, and that we were “adding even though we [were] subtracting.” They were very excited to see this change of events. I asked them to keep thinking about these numbers, the patterns, and why this change happened. I assured them that we’d have more opportunities to look at these interesting negative numbers. They knew they needed to keep thinking about the “why,” and we then moved on to the multiplication mini-lesson I’d planned for that day.

This Count Around the Circle routine, which is discussed in depth in Chapter 4, is a typical start to our math workshop. The predictability and ritualistic nature of routines in our classroom helps everyone feel at ease and participate, which promotes successful learning. Every day after our ten minutes of quiet time—which is our independent choice or rest time after lunch and recess to help us refocus for the afternoon—we come to the rug and sit in a circle. When I say, “Today we will Count Around the Circle,” or “Today we will play with quantities on the ten-frames” (discussed in Chapter 3), students know what that particular ritual entails. We know what to do. We know what to expect. It’s a comfortable and successful start to our math workshop each day.

In addition, the daily routine time gets students actively involved as they review number sense concepts and play with new number sense ideas; it also allows teachers time for formative assessment. In our case on that day, students had an opportunity to practice their estimation skills, practice counting backward by tens and twenties, continue to notice patterns in place value when counting, and begin exploring new ideas about negative numbers.

Number sense routines are a form of practice, but they are deep, meaningful practice. They serve to reteach, reinforce, and enrich. They maximize our time with students because they allow us to give our students multiple opportunities to strengthen and develop number sense. I find that number sense routines work most effectively when they occur at the same time each day. The predictable structure helps students make connections among routines from one day to the next. For example, in my fourth-grade classroom, we discussed the reasonableness of estimates before counting around the circle, because in the days prior, students’ estimates were often way off or they were trying to calculate rather than estimate. Sometimes their estimates didn’t fit with an obvious pattern. For instance, some students were making odd number estimates for a counting sequence that involved counting by twos starting at an even number.

In the days that followed this counting example, students started figuring out that Nisaa’s estimates were frequently *really* close to the actual answer (without calculating it exactly). They started paying more attention to her estimation strategies and tried to figure out why her estimates were reasonable and so close to the exact answer without calculation. With the repeated experiences, my students made connections from one day to the next and were really figuring out what it means to estimate. As they made those day-

to-day connections about estimation, they were also practicing a variety of counting sequences each day we did the routine. This practice over time helped my students gain understandings of relationships among numbers on the number line and notice patterns in place value. The children applied these understandings to their computation strategies and skills. The fluency with counting and the understanding of place value helped them become better and more efficient problem solvers. Lastly, the counting sequence in this example led the children to explore new ideas about negative numbers. The benefits of the counting routine during those weeks were deep, meaningful, and varied.

The number sense routines explored in this book are “responsive” routines—they are responsive to students’ discussions, understandings, and learning needs. All of the routines in this book do the following:

- Provide daily number sense experiences
- Include discussion about numbers and their relationships
- Respond to students’ current understandings
- Build on students’ existing number sense
- Encourage students to play with numbers and enrich their mathematical thinking
- Help students make connections to big ideas in mathematics

In other words, number sense routines provide a daily framework for number sense practice, yet these routines are responsive to students. They are fluid and flexible. In *The Morning Meeting Book*, Roxann Kriete and Lynn Bechtel say, “There is a sensitive balance between the lovely sense of security that routine can provide and the monotony that can creep in when that routine is unlivened and unleavened” (2002, 29). Routines provide a comfortable predictability, but at the same time, we plan routines that will keep students challenged, provide opportunities to practice using their number sense, and reteach when necessary.

WHY FOCUS ON ROUTINES?

If your classroom is similar to the average classroom nationwide, chances are that the range of learners is wide, from the student struggling with number sense to the student who continually needs a challenge. More and more teachers are using a math workshop format to meet the diverse needs of their students. There are many ways to set up a math workshop. Some teachers structure them as follows:

- Warm-up (or math message or ten-minute math)
- Mini-lesson

- Guided math groups
The teacher meets with small groups of four to five students while other students work in math stations, work on projects, problem solve, or work on a math game.
- Reflection or share

Other teachers set up their math workshops like this:

- Warm-up
- Mini-lesson
- Active learning or guided practice
Students work on an activity or some sort of problem or game related to the mini-lesson while the teacher confers with individual students or groups of students.
- Reflection or share

This book focuses on one component of your math block—your warm-up, which I refer to as a number sense routine, prior to the mini-lesson. Students need quick, explicit, daily experiences with number sense concepts. Routines provide that structure, no matter what you are teaching during the mini-lesson or during the active learning portion of the math block. The routine does not always need to be related or connected to the math lesson for that day or the math unit for that month. Its purpose is to provide a daily experience with a number sense concept. The ultimate goal is that students make connections over time, build an understanding of relationships among numbers and operations, and ultimately apply their number sense understandings in problem solving.

STUDENTS MAKING CONNECTIONS, UNDERSTANDING RELATIONSHIPS, AND APPLYING THEIR NUMBER SENSE: THE POWER OF NUMBER SENSE ROUTINES

Numerical literacy is the goal. We want students to build number sense and *use* their number sense. I keep my eye on that goal by observing students' number sense growth, then watching for its application to mathematics problems and discussions. All students have their own path as they move toward numerical literacy. Let me share snippets of Jaime's, Margaret's, and Andy's paths.

Jaime

One morning in early May, I began our math class with the following scenario: "We've been working outside in our garden to get it ready for planting. We've

talked about how much space we need between seeds. We've also observed other gardens with nice neat rows of plants. We decided our garden will need three rows of five seeds. How many pumpkin seeds will we need?"

Jaime, a first grader, started to solve the problem as he usually did, by directly modeling the situation. He took out the pumpkin seeds one by one and lined up five seeds, then lined up another row of five seeds, and finally lined up a third row of five seeds. After his seeds were organized into three rows of five seeds, he counted all the seeds by ones: "One, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15." However, this time Jaime did something different after counting by ones.

"Hey, that's like 5, 10, 15, like the red and white beads!" he said, referring to the rekenrek, a Dutch arithmetic tool we had been using earlier to explore the power of the five- and ten-structure of numbers (discussed in depth in Chapter 3). This routine provided a visual for Jaime that helped him "see" how numbers are composed. Now he was able to recount the seeds by fives and pointed to each row, saying aloud, "Five, 10, 15, see!'" This was the first time Jaime had applied a more efficient counting strategy to solve a story problem.

This was a great moment for Jaime. Previously he had listened to other students solve problems using the more efficient counting strategy and was able to explain what they did, but he had never applied this more efficient strategy on his own. The daily interaction with number sense ideas affected the way Jaime began to think about numbers. He started to "see" quantities and apply his understanding to solving a problem, thereby becoming more efficient and fluent in his computation. This was a student who was "struggling with number sense." He had strategies to solve problems, but found it difficult to be more efficient. The key in Jaime's evolution of number sense was the daily engagement in routines—and for this specific example, it was the routine of visualizing quantities of fives by using the rekenrek routine and then using this visualization to solve a math problem more efficiently. In other words, the routines allowed Jaime eventually to use his number sense understandings and apply them to a mathematical situation.

Margaret

Like Jaime, Margaret was a student who was struggling. Unlike Jaime's distinct "aha" moment, Margaret's understandings and application of number sense developed over a longer period of time.

It took a while for Margaret to understand Count Around the Circle (discussed in depth in Chapter 4). Eventually, though, this ended up being the routine that helped her develop the confidence to attack even the most daunting math problems. By midyear in third grade, after participating in numerous whole-class and small-group Counts Around the Circle, I noticed that Margaret was finally becoming more fluent with a variety of counting

sequences, even when we started at various points (for example, counting by hundreds starting at 347). When asked, “What do you notice about this counting sequence?” she was able to discuss which place in the number was changing and why. This new fluency in counting and understanding of place value was also transferring to her problem solving during the rest of math workshop. She began counting by tens and hundreds rather than by ones. The first time I watched her count backward by hundreds to solve a subtraction problem (*There were 783 books at the book fair and Ms. Lindgren sold 200 of them*), I wanted to jump up and down with joy.

Count Around the Circle had helped her develop a mental number line, understand the patterns of our place-value number system, and use leaps of friendly numbers like 100 and 10 to solve problems. Her ability to problem solve and her confidence in solving math problems skyrocketed as she “got” Count Around the Circle.

Andy

Unlike Jaime and Margaret, Andy had a fairly strong sense of number when he entered my classroom in the fall of his third-grade year. He knew how to decompose numbers and use his understandings of place value to solve problems using tens and ones. He knew how to skip-count by a variety of numbers, which helped him solve multiplication and division problems.

What I observed during our routines (mostly during our discussions about the mathematics in Count Around the Circle and Quick Images with dot cards) was that Andy was growing his number sense in terms of relational thinking. During the number sense routines, Andy was pointing out relationships he noticed among numbers and equations. When we looked at Quick Images with dot cards he eventually started playing with ideas of equality, the distributive property, and the associative property, stating, “Four groups of 3 is the same thing as 2 groups of 6. It just depends on what the story problem is, but you can just take those 2 groups of 3 and make them 6 and take the other 2 groups of 3 and make that 6—either way the total is 12.” (See Andy’s thinking in Figure 2.2.)

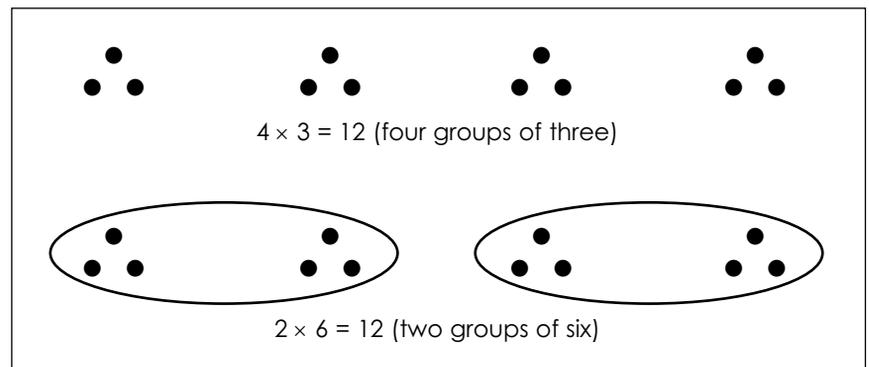


Figure 2.2
Andy's Thinking About Quick Images

Andy practiced his relational thinking as we worked on true/false statements like those that follow. (Note: true/false statements are a number sense routine that is not discussed in this book. For more information, see *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School* by Thomas P. Carpenter, Megan Loef Franke, and Linda Levi [2003].)

$5 + 4 + 10 = 10 + 5 + 5$	false
$13 + 7 + 4 - 4 = 7 + 13$	true
$13 + 9 + 6 = 5 + 10 + 13$	true
$6 + 3 + 10 = 8 + 3 + 7 + 2$	false

I observed that he was thinking more and more about relationships among numbers rather than just solving for each side. For example, for the second equation in the preceding list, Andy said he didn't solve both sides because he knew "Four minus four is zero, so that balances the equation."

The more that Andy was thinking relationally during our number sense routines, the more I saw him apply that understanding to other problems. Not only did I watch him use a compensation strategy (which uses relational thinking) in math workshop but I also heard him state a direct connection to the problems we used in our true/false statements: "I know 98 plus 37 is 135 because I moved some of the numbers around like we do in the true/false number sentences. I know that 98 is only two away from 100, and 100 is easier to work with, so I took 2 from the 37 and made it 35. That way 100 plus 35 is 135, and that was easier than 98 plus 37, but it's okay because they mean the same thing." (See Figure 2.3.) Andy was applying what he knew about relationships among numbers and equations to problems such as these. He was becoming more numerically literate due to the variety of routines he experienced each day.

• • •

The power of routines that provide students opportunities to interact with numbers, big math ideas, and strategies on a daily basis is exemplified in students like Jaime, Margaret, and Andy. Building number sense every day through routines will improve students' numeracy. By using predetermined sets of routines to enhance the experience in a creative math environment, the teacher sets the stage for successful development and use of number sense.

A LOOK AHEAD

Part II of this book explores ideas for routines I have found to be most effective in helping students build a strong sense of number. The following tables list and summarize the routines that will be discussed in the following chapters.

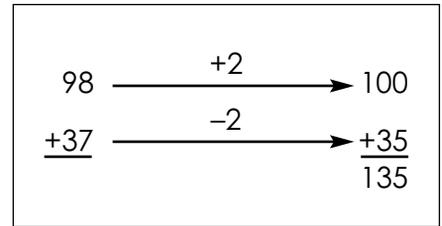


Figure 2.3
Andy's Thinking About $98 + 37$

Chapter 3 **Visual Routines: Seeing and Conceptualizing Quantities**

Name of the Routine	Helps with . . .	How It Works	Ways to Use the Routine and Questioning Strategies
Quick Images Using Dot Cards (and Pictures, Dominoes, and/or Dice) (page 36)	<ul style="list-style-type: none"> • Subitizing • Visualizing amounts • Using groups and combining groups to figure out “how many” 	<p>These are cards with dots on them arranged in various groups. You can make your dot cards based on twos, fives, tens, doubles, or the visual arrangement of dice or dominoes. You flash the amount quickly, giving students about 3–5 seconds to visualize the amount. Then, you ask students what they saw. This will encourage them to think in groups rather than count by ones.</p>	<p>To elicit thinking about Quick Images, ask these questions:</p> <ul style="list-style-type: none"> • <i>How many did you see?</i> • <i>How did you know it so quickly?</i> • <i>Did you need to count? So what did you do? What did you see?</i> • <i>Why are you able to know the amount so quickly?</i> <p>To discuss <i>perceptual subitizing</i>, use the following:</p> <ul style="list-style-type: none"> • 3 dots: <i>Did you count each dot or did you just see the amount?</i> • 5 dots: <i>Did you count? Did you see an amount?</i> (Some students might see the 5 as a whole amount; others may see 3 and 2 or 4 and 1.) • 3 dots and 1 dot: <i>How many dots? How did you see it?</i> • 2 dots and 2 dots: <i>How many dots? How did you know?</i> <p>To encourage <i>conceptual subitizing</i>, use the following:</p> <ul style="list-style-type: none"> • 2-by-2 array with 2 dots off to the side: <i>How many dots? How did you figure it out?</i> • 5 dots in dice formation with 4 dots in dice formation: <i>What did you do to figure it out quickly?</i> • A card arranged with 1 dot, 2 dots, and 3 dots: <i>How many dots? How did you combine the dots to know how many?</i> • 4 rows of 3 dots: <i>How did you know the total so quickly?</i>
Ten-Frames (page 43)	<ul style="list-style-type: none"> • Grouping • Using the ten-structure and five-structure • Composing and decomposing ten • Teen numbers • Part-part-whole ideas 	<p>You can use the ten-frame much like Quick Images. The difference in using the ten-frame is that the five- and ten-structures are highlighted by the configuration of the frame.</p> <p>The ten-frame can better highlight the idea of teen numbers—the</p>	<p>To elicit thinking about ten-frames, ask the following:</p> <ul style="list-style-type: none"> • <i>How did you figure out how many?</i> <p>To work on <i>combinations of ten</i> and the <i>commutative property</i>, use problems like these:</p> <ul style="list-style-type: none"> • $9 + 1$ and $1 + 9$, $8 + 2$ and $2 + 8$, $7 + 3$ and $3 + 7$, etc.

Chapter 3 Visual Routines: Seeing and Conceptualizing Quantities (continued)

		<p>concept that a teen number is a ten and then some more.</p> <p>The ten-frame can also be used for two-digit addition and subtraction.</p>	<p>To work on <i>teen numbers</i>, use the ten-frame to discuss and figure out amounts like this:</p> <ul style="list-style-type: none"> • Fourteen is composed of a full ten-frame plus a ten-frame with 4 dots. <p>Children can use ten-frames to practice <i>addition</i> with problems like this:</p> <ul style="list-style-type: none"> • A ten-frame with 9 dots plus a ten-frame with 4 dots: Children will often move 1 dot from the 4 to the ten-frame with 9 to make 10, then do $10 + 3$. <p>To explore <i>part-part-whole relationships</i>, use problems like this:</p> <ul style="list-style-type: none"> • Show a ten-frame with 6 dots. Ask: <i>How many dots are needed to make 10?</i>
Rekenrek (page 49)	<ul style="list-style-type: none"> • Grouping • Using the ten-structure and five-structure • Composing and decomposing 20 (or 100 on the rekenreks with 100 beads) • Teen numbers • Part-part-whole ideas 	<p>The rekenrek is a Dutch arithmetic rack. It has two rows with 10 beads on each (or, on a rekenrek with 100 beads, ten rows with 10 beads on each row). Each row of 10 beads is made up of 5 red beads and 5 white beads. There is a white panel attached to the end of the frame that allows you to hide some beads and show other beads.</p> <p>You can use the rekenrek in a Quick Images manner to encourage the use of groupings. And, like the ten-frame, the rekenrek highlights the five- and ten-structures. The rekenrek is different in that it has 20 beads total (or 100 beads total) and the beads move on the rods, giving it a kinesthetic aspect.</p>	<p>Use these questioning strategies with the rekenrek:</p> <ul style="list-style-type: none"> • <i>Can you show a way to make fifteen? Can you show another [a different] way to make fifteen?</i> • <i>How many do we need to add to make seventeen?</i> • <i>How many do we need to take away to make twelve?</i> • <i>What can we do to make eight?</i> • <i>How many are hiding behind the white panel?</i>

Chapter 4 **Counting Routines: Understanding Place Value and the Number System**

Name of the Routine	Helps with . . .	How It Works	Ways to Use the Routine and Questioning Strategies
Count Around the Circle (page 57)	<ul style="list-style-type: none"> Counting sequences Using patterns for problem solving Estimation Understanding place value Understanding how the number system works 	Choose a counting sequence—for example, count by tens starting at thirty-two—and go around the circle as each person says a number. (For example, the first person says, “Thirty-two,” the second person says, “Forty-two,” the next person says, “Fifty-two,” and so on.)	<p>Variations on this routine include the following:</p> <ul style="list-style-type: none"> Count by ones, tens, fives, twos, threes, and so on, starting at zero. Count by ones, tens, fives, twos, threes, and so on, starting at various numbers. Count by fractional numbers. Count by hundreds or thousands or millions, starting at zero or at various numbers. <p>To facilitate understanding of the patterns, write the numbers on the board as students say them.</p> <p>Ask a variety of questions to differentiate the level of difficulty. (For a list of questions, see Box 4.3).</p>
Choral Counting (page 66)	<ul style="list-style-type: none"> Counting sequences Understanding patterns in numbers 	In this routine, the class counts aloud a number sequence all together.	<p>Use this routine if the majority of the class is struggling with the counting sequence.</p> <p>Use a number grid or number line as students are counting to help students see and use the patterns. (See the appendix for various versions of number grids.)</p> <p>To facilitate higher-level thinking and spark discussion about the sequence, ask: <i>What do you notice about this pattern?</i></p>
Start and Stop Counting (page 67)	<ul style="list-style-type: none"> Counting sequences Understanding patterns in numbers Difference or distance between two numbers 	The class counts a number sequence all together, with a starting number and a stopping number. For example, have the class count by tens, starting with 26 and stopping at 176. In addition to whole class, this routine works particularly well with small groups and individual students.	<p>Ask questions to facilitate discussion about <i>patterns</i>, such as odd/even patterns:</p> <ul style="list-style-type: none"> <i>If we start with twenty-five and count by fives, what numbers could we stop at?</i> <i>If we count by twos and start with 1,222, what numbers could we stop at? Why would the number need to be even?</i> <p>To highlight the <i>distance between numbers</i> and guide a discussion</p>

Chapter 4 **Counting Routines: Understanding Place Value and the Number System** (continued)

			<p>about <i>difference</i>, use the following questions:</p> <ul style="list-style-type: none"> • If we count by twos, starting with 1,222 and stopping at 1,234, will it take a long time or not much time? How do you know? • If we count by twos, starting with 1,222 and stopping at 4,222, will it take a long time or not much time? How do you know?
Organic Number Line (page 72)	<ul style="list-style-type: none"> • Irrational numbers • Various names and representations of numbers • Big ideas like benchmarks, equivalence, the whole, and part of the whole • Strategies like using benchmarks and doubling and halving 	<p>This is a number line that you can add to continuously throughout the year. Think of it as one section of your “whole number” number line—you are magnifying (and hence adding more details to) the number line from 0 to 2. For example, there are many numbers that fall between 0 and 1: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, 0.25, 0.3333, etc. There are also different ways to represent each of these numbers, and some of these numbers are equivalent.</p>	<p>To focus on <i>benchmarks</i>, ask questions like these:</p> <ul style="list-style-type: none"> • Where does this number go on our number line? How do you know? • What numbers can you think of that go between $\frac{1}{2}$ and 1? How do you know? <p>To focus on <i>equivalency</i>, use prompts and questions like these:</p> <ul style="list-style-type: none"> • Prove that $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent. • Can you show another way to represent $\frac{4}{16}$? <p>To focus on <i>the whole and parts of the whole</i>, ask questions like this:</p> <ul style="list-style-type: none"> • Are this half and this half the same amount? (Show two models representing $\frac{1}{2}$, but each with a different whole.) Prove it! <p>To focus on <i>doubling and halving</i>, ask questions like this:</p> <ul style="list-style-type: none"> • What is half of $\frac{1}{4}$? Where does that fraction go on the number line?

Chapter 5 **Playing with Quantities: Making Sense of Numbers and Relationships**

Name of the Routine	Helps with . . .	How It Works	Ways to Use the Routine and Questioning Strategies
Ten Wand (page 81)	<ul style="list-style-type: none"> • Combinations of ten • Commutative property • Part-part-whole ideas • Ten-structure and five-structure 	<p>The Ten Wand is made up of ten Unifix cubes, five of one color and five of a different color. The wand breaks in two pieces at various places (decomposing the ten) to help students see combinations visually.</p>	<p>Use questioning strategies like these when working with the Ten Wand:</p> <ul style="list-style-type: none"> • <i>How many on the floor and how many in my hand?</i> • <i>How did you see seven so quickly? How did you know that's seven without counting it?</i> • <i>What is it about the wand that made it easy to see the amount?</i> • <i>If we put the parts back together, how many cubes make up the wand now? Why is it still ten?</i> • <i>So if there are two on the floor, how many more are needed to complete the broken wand?</i>
Ways to Make a Number (page 83)	<ul style="list-style-type: none"> • Thinking flexibly about numbers • Composing and decomposing numbers • Place-value understanding • Base ten and grouping ideas • Relationships among ones, tens, and hundreds 	<p>Students write as many ways as they can think of to “make” a selected number. They might use visuals of the quantity, equations, models, and so on.</p>	<p>This routine can be open-ended (just give students the number and no guidelines) or it can have constraints (such as, <i>Think of ways to make this number with three addends</i>).</p> <p>Use questions like these with Ways to Make a Number:</p> <ul style="list-style-type: none"> • <i>What is it about ten that gave you the idea to write it that way?</i> • <i>Why does that work?</i> • <i>How do you know it works?</i>
Today's Number (page 88)	<ul style="list-style-type: none"> • Understanding numbers embedded in various contexts • Numbers' relationships to 10 and 100 • Grouping ideas (repeated groups, base ten, tens bundled as a hundred) 	<p>The teacher chooses a number, such as ten, to be Today's Number (there are a variety of reasons for picking a particular number) and asks various questions about the number, such as: <i>When is ten big? When is ten small?</i></p>	<p>In order to help students understand numbers in various contexts, ask questions like these:</p> <ul style="list-style-type: none"> • <i>When is ten a large amount?</i> • <i>Why did you think of that as an example of when ten is a large amount?</i> • <i>When is ten not very much?</i> • <i>Why does ten mean different things in different contexts?</i> <p>(See Box 5.3 for a complete list of ideas and for questions to use with Today's Number.)</p>

Chapter 5 **Playing with Quantities: Making Sense of Numbers and Relationships** (continued)

			<p>To highlight a number's relationship to 10 and/or 100, ask questions like these:</p> <ul style="list-style-type: none"> • How far is 24 from 100? How do you know? • How far is twenty-four from ten? How did you figure it out? • How many 24s are in 100? <p>Use the following questions to elicit discussion about base ten ideas in relation to Today's Number:</p> <ul style="list-style-type: none"> • How much is ten groups of twenty-four? • How many tens are in twenty-four?
Mental Math (page 91)	<ul style="list-style-type: none"> • Efficient strategies • Flexible thinking • Place-value understanding • Base ten and grouping • Using relationships among numbers • Computation and operations properties 	Present an equation or story problem and ask students to solve it in their heads (without paper and pen or manipulatives). Children should then verbalize the strategies they used mentally.	<p>To facilitate students verbalizing their mental math, use questions like these:</p> <ul style="list-style-type: none"> • What did your brain do? • Why does that work? • Who can restate what Kelly said/did in her head? Why do you think she used that strategy? • What part was tricky to do without paper?

Chapter 6 **Calendar and Data Routines: Using Numbers Every Day**

Name of the Routine	Helps with . . .	How It Works	Ways to Use the Routine and Questioning Strategies
Calendar (page 102)	<ul style="list-style-type: none"> • Understanding how our time is organized and measured • Counting, recognizing, and sequencing numbers 	Use a real calendar in addition to a premade calendar from the teacher store. As a class, write in important days throughout the school year (birthdays, field trips, etc.). Integrate social studies and science.	<p>Questions to use for the calendar routine include these:</p> <ul style="list-style-type: none"> • <i>What is today's date? What was yesterday's date? What will tomorrow be?</i> • <i>How many days (or months) until Thanksgiving?</i> • <i>When did we go to gym class?</i> • <i>If January ends on a Monday, on what day will February begin?</i>
Collecting Data Over a Long Period of Time (page 104)	<ul style="list-style-type: none"> • Using numbers in authentic ways • Thinking about patterns and cycles • Getting a sense of measurement amounts • Using descriptive statistics 	Collect data, such as temperature, weather, and sunrise/sunset times, over time on graph paper in public spaces in the classroom on a daily basis. Once or twice a month, hold class discussions about the data trends and the interpretation and analysis of the data.	<p>Discuss <i>patterns in temperatures and weather</i> with questions like these:</p> <ul style="list-style-type: none"> • <i>What do you notice about the data? What tells you that?</i> • <i>What do you think this graph will look like next month? How do you know?</i> <p>To encourage the <i>use of descriptive statistics</i>, ask questions such as these:</p> <ul style="list-style-type: none"> • <i>What is the most common temperature this month?</i> • <i>What is the most common type of weather this month?</i> • <i>What is the mean temperature in January?</i> • <i>What's the range in temperature for September? How is it different from the range in December?</i> <p>Examine the <i>visual pattern of sunrise and sunset times</i> and ask questions such as these:</p> <ul style="list-style-type: none"> • <i>What do you notice about the length of the day over time?</i> • <i>What patterns do you notice in the data?</i>
Counting the Days in School (page 109)	<ul style="list-style-type: none"> • Gaining a sense of growing quantities • Keeping track of information • Thinking about patterns 	Use sentence strips and sticky notes to build a number line throughout the year that will emphasize each tenth day of school.	<p>To help students have discussions about the growing quantities and the patterns in keeping track of the days in school, ask questions like these:</p> <ul style="list-style-type: none"> • <i>What color sticky note do you need for today? How do you know?</i> • <i>How did you know what number comes next?</i>

Chapter 6 **Calendar and Data Routines: Using Numbers Every Day** (continued)

	<ul style="list-style-type: none"> Beginning to think about why ten is an important and friendly number 	<p>Use a number grid from 1 to 180 to keep track of the days in school.</p> <p>Add one cube to a container each day you are in school (and eventually organize the cubes into tens to count efficiently).</p> <p>Add one rock to a container each day you are in school (see a pile grow).</p>	<ul style="list-style-type: none"> <i>How many days will it be on Friday?</i> <i>Which number on the number grid will we move the circle to on Friday?</i> <i>How many days until the 100th day of school? How do you know?</i> <i>How will you count the cubes?</i> <i>About how much of the rock jar do you think will be filled up by the seventy-fifth day of school?</i>
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