Study Guide

COMMON CORE SENSE

Tapping the Power of the Mathematical Practices

Christine Moynihan

Contents

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Overview

Since the arrival of the Common Core State Standards, states that have adopted the CCSS are working hard to "unpack" them. This is a process that takes both time and sustained support for teachers first to make sense of the standards and then to implement them in a reflective and meaningful way.

Common Core Sense: Tapping the Power of the Mathematical Practices is meant to be a vehicle for making the eight Standards for Mathematical Practice more accessible to elementary teachers. Christine Moynihan sees the Mathematical Practices (MPs) as the core of mathematical proficiency and identifies the purpose of this book as a way to help teachers tap into their inherent power. Although teachers have seen the list of the Mathematical Practices, many report that they have not had much of an opportunity to explore them deeply. To that end, the author shares a framework to help teachers gain a stronger foothold in their knowledge and understanding of the MPs and provides a glimpse into how they may come to life in a classroom.

It is the author's hope that, as a result of using what she calls the GOLD framework, teachers can break apart each of the MPs and think about what each may look and sound like in classrooms, and then help them think about what needs to be done to support the incorporation and implementation of the MPs into daily practice.

The Framework

Go for the goals: What are the major purposes of the practice?
Open your eyes and observe: What should you see students doing as they use the practice? What should you see yourself doing as a teacher?
Listen: What should you hear students saying as they use the practice? What should you hear yourself saying?
Decide what you need to do as a teacher: What actions must you put in place in order to mine the gold of the practice?

What follows is a study guide designed to help groups of teachers share new thinking about the Mathematical Practices. In this way, teachers can support one another as they tackle the MPs, discuss them, reflect upon them, and then try different techniques and strategies with their students. Change, after all, is a process, one that is often made infinitely easier by sharing with companions on a similar journey—that of bringing students to new heights of mathematical proficiency and new depths of understanding.

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Chapter 1—Mathematical Practice 1: Make Sense of Problems and Persevere in Solving Them

Questions for Group Discussion

- What does this MP mean to you?
- What do you think *problem solving* means to your students?
- Why can it be a good idea to give students the answer to a problem before they work to solve it?
- Why do you think it is hard for many students (and even adults) to persevere?

Quotes to Ponder

- Being a problem solver means that one is able to analyze problems, reason about them, build arguments that support solutions, connect them to everyday life, use the right tools at the right moment to solve them, and be precise in communicating how they can be solved while at the same time looking for and using patterns and structures that are regular and repeat. (7)
- We are well served by committing many things to memory, but being a good memorizer does not necessarily make one a good problem solver. (7)
- I make a cautionary note that the learning of specific strategies should be seen as a means to an end, not the goal itself, thus supporting the idea that strategies should be viewed as “powerful tools for mathematical thinking” (Chapin, O’Connor, and Anderson 2009, 91; page 12 in my book).
- Self-efficacy makes a difference, as “self-efficacious students show greater perseverance during adversity, are more optimistic, have less anxiety, and achieve more than do students who lack self-efficacy” (Rollins 2014, 121; page 12 in my book).

Activities for Tapping the Power of MP1

- Solve the problem “Bicycle Business” independently. Make notes about the strategy(ies) you use. Also take note of what was easy for you and what was more challenging. Discuss with your colleagues and identify for each of you what elements of MP1 were evident.
- Think about the strategies you use to solve problems; list three of your favorites and identify why you most often default to them. Share and discuss with your colleagues.
- Actively look for “good” mistakes made by your students. Think about what made them good. Reflect on how you took advantage of them and how you could expand upon their untapped power.
- Spend a bit of time reading about self-efficacy. Share your findings with your colleagues. Collectively create a list of deliberate teacher actions you could put into place to foster the growth of self-efficacy in your students.

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Chapter 2—Mathematical Practice 2: Reason Abstractly and Quantitatively

Questions for Group Discussion

- What are the differences between abstract and quantitative reasoning?
- What does *number sense* mean to you, and why is it so important?
- Why do you think it is often challenging for students to extract the meaning from a contextualized problem (a "word" problem) and represent it quantitatively? What can you do to support them?
- How does the saying, "If you give a man a fish, you feed him for a day; if you teach a man to fish, you feed him for a lifetime," apply to teaching? Why is it often hard for teachers to resist just "giving the fish" to our students?

Quotes to Ponder

- Many of us were robbed of the chance to learn at an early age how to reason both abstractly and quantitatively. MP2 is asking us not to allow that to happen to this generation of students. (23)
- In the long run, however, we all recognize that once students accept that somewhere in the jumble of the words in a problem there lies a way to make sense of the problem and build a model of it, and that they are capable of finding it with some scaffolding and practice, they are better served both in the moment and in the future. (24)
- [Fennel and Landis], like many others, urge elementary teachers especially to understand that a strong number sense equals a strong foundation for reasoning and that this is a component essential to mathematical proficiency through middle school, high school, and beyond. (26)
- "One of the most interesting aspects of Standard for Mathematical Practice 2 is the emphasis on students being able to move back and forth while solving a contextual problem between a situation and the mathematical representation of the situation" (Seeley 2014, 273; page 28 in my book).

Activities for Tapping the Power of MP2

- Look at the student work samples in this chapter for "Heads, Shoulders, Knees, and Toes." If it is appropriate for your grade level, give this problem to your students and analyze their work in the same way. Bring samples of student work to analyze with your colleagues.
- Discuss with your colleagues what goes into a strong "think-aloud," where one of you thinks through a written problem aloud while working to represent the problem quantitatively. Role-play with each other and dissect the results.

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- Create a shared definition of *number sense*. Identify a few core activities that support the development of number sense.
- Cathy Seeley (2014) talks about "zooming in and zooming out" as students move back and forth from a mathematical situation and its quantitative representation. Discuss what this might look like in your classrooms.

Chapter 3—Mathematical Practice 3: Construct Viable Arguments and Critique the Reasoning of Others

Questions for Group Discussion

- What separates an opinion from a valid argument? Which do you think is harder to defend, and why?
- Why is *Why?* such an important word in the classroom? What steps can you take to make it more prevalent, more substantive, more valued?
- Why is it often difficult for students (as well as for adults) to accept feedback as something positive? What can you do to make it feel safer for students to receive feedback?
- What does it really mean to "critique" the reasoning of someone else? How would you evaluate your students' collective ability to do this?

Quotes to Ponder

- NCTM (2000, 60) maintains that when our students "are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing," which moves the communication from an opinion to a valid argument. (41)
- Simply put, there are *no* tricks in mathematics, and there is no magic in mathematics (other than the way it can create beauty and wonder). (42)
- Many times students believe that simply listing the steps they used to solve a problem is a way to explain and justify their thinking. In reality, however, telling and showing ≠ proving and justifying. (43)
- When students perceive the classroom community as safe, they are more likely to share their reasoning and strategies, even if they are not completely formed. (46)

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Activities for Tapping the Power of MP3

- Solve "The Inheritance" problem independently. Try to solve it more than one way. Be ready to share and justify your thinking with your colleagues. Take note of the similarities and differences in the various ways to solve this problem.
- Bring samples of student arguments for all of your colleagues. Tease out the strengths of each and the next steps you would take.
- Read more about how to structure and support mathematical discussions (see Kazemi and Hintz 2014).
- Work as a group to identify common "tricks" your students come to you already knowing and/or ones that are taught to them by well-meaning parents. Determine why they may technically work but also identify the understanding they circumvent. Discuss how to replace them.

Chapter 4—Mathematical Practice 4: Model with Mathematics

Questions for Group Discussion

- What does *mathematical modeling* mean to you? Has your view of it changed since reading this chapter? If so, how?
- When do you use mathematics in your everyday life? How do you use it? What kinds of decisions can it help you make?
- What do you think the term *messy problems* means? Why do you think it is important for students to have experiences with these types of problems?
- How does "using what you know to find what you don't know" apply to this MP?

Quotes to Ponder

- One way for students to see the value of the mathematics they are learning is to have them work with situations in their everyday lives that can be "mathematized." (58)
- To manipulate these models, however, students must have experiences in which they have had to meld conceptual understanding with procedural knowledge, so they will avoid what John Tapper calls a student facing a "dichotomy between *knowing* the steps and *understanding* the steps" as something akin to "having a list of directions to get where you are going versus having a map of the entire area" (2012, 14; page 60 in my book).
- I find that a fairly uncomplicated strategy—make it simpler—is not apparent enough in [students'] problem-solving repertoire to be used as often as it could and should be.

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Students need to be taught explicitly a variety of ways to make a problem simpler. (60)

- "Generalization, it can be argued, is the goal of all learning. It means that students can use what they learn in class in a variety of new, unfamiliar circumstances. It is proof that students have incorporated new concepts and can use them—rather than learning them for the purpose of producing them for the teacher" (Tapper 2012, 155; page 60 in my book).

Activities for Tapping the Power of MP4

- Look at the student work samples in this chapter for "Show Me the Value." If the problem is an appropriate one for your grade level, try it with a few students or the whole class (modifications can be made easily). You may even try it yourself. Try to identify how this can highlight some of the basic components of MP4. Discuss with your colleagues.
- Explore the relationship and connections that exist between this MP and MP1, MP2, and MP3.
- Look for instances in your classroom that show the difference between a student *knowing* the steps and *understanding* the steps to solve a problem. Discuss with your colleagues.
- Collectively create a list of what makes a problem "messy." Start building a collection of messy problems to be shared.

Chapter 5—Mathematical Practice 5: Use Appropriate Tools Strategically

Questions for Group Discussion

- What are the mathematical tools of the elementary classroom? (Be sure to move beyond concrete tools.)
- What are the purposes of mathematical tools?
- How do you determine whether a tool has become a crutch for some students? What can you do about it?
- Why is it important for students to have opportunities to change their minds about which tool they have chosen to complete a mathematical task?

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Quotes to Ponder

- Exposing students to a variety of tools and letting them use them to explore can open the doors to problem solving even wider. (80)
- Just because a tool can do certain things and we enjoy using it does not mean that it is the right one for a particular job. (80)
- "If students use tools to engage in mathematics and walk away from the experience with little or no understanding of the mathematics, then the use of the tools was ineffective" (Kanold 2012, 46; page 82 in my book).
- The use of tools in mathematics can support such pondering on the parts of our students and makes it easier for them to pose questions, to run with their curiosity, to imagine. (84)

Activities for Tapping the Power of MP5

- Look at the student work samples in the chapter for the "Let It Snow" problem. Take note of which tools students used and how they used them. Consider whether you would add other tools to be used. What mathematics do you think these students know? What do they not know?
- Take an inventory of your classroom collection of concrete mathematical tools. Determine whether they are accessible to all students, whether they are organized optimally, whether they are appropriate for your developmental span, etc.
- Discuss with your colleagues strategies to help those students who are reluctant to use concrete tools because they see the use of tools as a sign of mathematical weakness.
- Collectively create the components of a "mathematical backpack" that includes concrete, pictorial, and symbolic tools for your grade level.

Chapter 6—Mathematical Practice 6: Attend to Precision

Questions for Group Discussion

- What does *mathematical precision* mean to you? Discuss how your definition may have broadened as a result of reading this chapter.
- How do you scaffold mathematical language development?
- How much time do you provide for students to "talk math"?
- In your everyday life, do you more often need an estimate or an exact answer? When you need an exact answer, how do you determine to what level of precision you need it? What kinds of answers are our students most often asked to find? Is there a mismatch here that requires attention?

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Quotes to Ponder

- We must help [students] see that an essential part of thinking mathematically lies in how we communicate and share that thinking, and that this is where precision is key. (95)
- As a matter of fact, research does more than suggest a simple link between language and learning, summed up quite nicely by Laura Varlas, who says, "Academic vocabulary is one of the strongest indicators of how well students will learn subject area content" (2012, 1; page 96 in my book).
- "By being careful with our own language and communication, we can also avoid 'temporary mathematics' that may need to be later undone" (Seeley 2014, 314; page 98 in my book).
- Although this support [to communicate] comes from us as teachers, I see peers as enormous sources of support and agree with Elham Kazemi and Allison Hintz, who maintain, "It can be quite powerful for a classroom community when students share ideas that aren't quite right yet and seek the help of their classmates" (2014, 12; page 100 in my book).

Activities for Tapping the Power of MP6

- Look at the student work samples of "My Restaurant" in this chapter. Identify the elements of precision that are visible. Look to see if there are missing components that would have improved student work.
- Collectively create a list of language "pitfalls" and "outlaws" that may be prevalent at your grade level. Discuss why each can lead to misunderstanding. Strategize how to replace each with more precise language.
- Have students create a "math word wall" for your classroom. Make it active by referring to it consistently and encouraging students to do the same.
- Work with your colleagues to create a list of essential mathematical vocabulary by grade level. Strategize ways to integrate it into daily instruction.

Chapter 7—Mathematical Practice 7: Look For and Make Use of Structure

Questions for Group Discussion

- What does *structure* in mathematics mean to you? How do you look for it?
- How can using structure be helpful in mathematics? What can it lead to in terms of learning?
- Is it easier for you to find and use numeric patterns or geometric ones? Why? What

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about for your students?
- How can "functional fixedness" be restrictive and limiting? How can you help students move beyond it?

Quotes to Ponder

- To discern a pattern, one must look closely at the underlying structure, and then step back to gain perspective, generate some ideas about the nature of the structure—the regularities that can be seen as well as envisioned beneath and within—apply those ideas, and then assess to determine if the pattern holds. (113)
- Something to remember here is that although it is important for students to have multiple opportunities to name, replicate, and extend patterns as a basis for laying the groundwork for algebraic reasoning, the full benefit of this will be actualized "only if we prompt children to attend to the mathematical properties and describe the repeating structures in mathematically predictable ways" (McGarvey 2013, 571; page 114 in my book).
- When students become aware of how efficient it is to use what is already in their backpacks of knowledge, so to speak, they experience greater success in solving more complex problems. (117)
- This prior knowledge, or background knowledge, is not only the "glue that makes learning stick," according to ReLeah Lent, but also "an essential component in learning because it helps us make sense of new ideas and experiences" (2012, 30; page 117 in my book).

Activities for Tapping the Power of MP7

- Solve the "Square Thinking" problem from this chapter independently. Step back and assess how you used or didn't use structure to help you. Be ready to share your thinking with your colleagues.
- Work with your colleagues to create a bank of problems that are rich in mathematical structure. Work through a couple of them as a group and discuss your observations.
- Ask your students what a pattern is and how it can be used. Collect their responses and share them with your colleagues. If the definitions are narrow, strategize collectively how to expand their thinking.
- Explicitly model how "using what you know to find what you don't know" is making use of structure.

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Chapter 8—Mathematical Practice 8: Look For and Express Regularity in Repeated Reasoning

Questions for Group Discussion

- What does *regularity in repeated reasoning* mean to you? Did your thinking about this change as a result of reading this chapter? If so, how?
- What are some of the benefits of using regularities in repeated reasoning? Think about some specific examples.
- Do you see the use of this MP as an important part of being mathematically proficient? Why or why not? What do you think the general public thinks of this?
- Where and how do you see this MP interconnect with MP1 and MP3?

Quotes to Ponder

- Harder and more complicated mathematics becomes simpler and more accessible through the application of the knowledge that comes by way of identifying and using regularities in repeated reasoning. (129)
- This does not mean that instructional time on computation is eliminated but that it is reduced. This can be accomplished with the result of students becoming proficient by having them notice and take advantage of the regularities so that they, in [Cathy] Seeley's words, "generalize the procedures they're learning so that they don't have to learn the same procedures over and over again." Seeley goes on to say, "In other words, we want to teach students a procedure that works for all kinds of numbers, not just for three-digit numbers this year, four-digit numbers the next year, and so on" (2014, 337; page 130 in my book).
- If we want our students to believe that mathematics is essential to their lives, that it is relevant to their lives now and will be in the future, that it is something that has meaning and does make sense, then we must immerse them in problem-solving opportunities that require them to find the meaning and the sense in what they are doing by way of their mathematical reasoning. (134)
- "For deep understanding, the what, the why, and the how must be well-connected. Then students can attach importance to different patterns and engage in mathematical reasoning" (Sousa 2008, 123; page 134 in my book).

Activities for Tapping the Power of MP8

- Complete the "Go Figure" problem from this chapter independently. Examine your solution in light of using regularity in repeated reasoning. Be ready to discuss your solution and reasoning with your colleagues.

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- Choose a similar problem to work through with your colleagues. Discuss ways to differentiate the problem for various levels. Spend time highlighting the regularities within the problem that can lead to more efficient problem solving.
- Work collectively to outline ways to infuse this MP into computational work in all four basic operations.
- Partner with a colleague and observe his or her classroom to look for use of this MP by both your colleague and the students.

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