IT'S ALL RELATIVE

Key Ideas and Common Misconceptions About Ratio and Proportion, Grades 6–7

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Introduction

Understanding ratios and proportional relationships and acquiring the accompanying skills associated with their conceptual development are essential. These ideas permeate our daily lives and underpin further study in mathematics and science (Common Core Standards Writing Team 2011). The Common Core State Standards for Mathematics (CCSS-M) identify this area of study as critical at both grades six and seven (NGA and CCSSO 2010).

The thirty modules in this flipchart are designed to engage all students in mathematical learning that develops conceptual understanding, addresses common misconceptions, and builds key ideas essential to future learning. The modules are research based and can be used to support response to intervention (RTI) as well as offer enrichment activities and challenges for all students. The modules are organized in three sections: Representing Ratios; Unit and Scale Factors; and Percents. While building on students' understanding of multiplication and division, the activities in this flipchart will focus on these key ideas:

- Understanding the language of ratios
- Understanding the multiplicative relationships of ratios
- Using tables, tape diagrams, double number line diagrams, and graphs to represent ratios
- Using unit rates and scale factors to solve problems
- Solving multistep ratio and percent problems

The modules increase in complexity by section, though we do not assume that you will focus on only one section at a time nor that you will necessarily complete each component of an activity or section. You can return to many of these activities as students build their mathematical expertise. Each activity begins with the identification of its **Mathematical Focus**, through identification of specific CCSS-M standards. (Either complete standards or portions thereof are provided.) The **Potential Challenges and Misconceptions** associated with those ideas follow. **In the Classroom** then suggests instructional strategies and specific activities to implement with your students. **Meeting Individual Needs** offers ideas for adjusting the activities to reach a broader range of learners. Opportunities to assess student thinking are often embedded within one section or another. Each activity is supported by one or more reproducibles (located in the appendix), and **References/Further Reading** provides resources for enriching your knowledge of the topic and gathering more ideas.

We encourage you to keep this chart on your desk or next to your plan book so that you will have these ideas at your fingertips throughout the year.

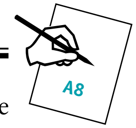
REFERENCES/FURTHER READING

Collins, Anne, and Linda Dacey. 2010. *Zeroing in on Number and Operations: Key Ideas and Common Misconceptions, Grades 7–8*. Portland, ME: Stenhouse.

Common Core Standards Writing Team. 2011. *Progressions for the Common Core State Standards in Mathematics: 6–7, Ratios and Proportional Relationships*. Draft. http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf.

National Governors Association (NGA) and Council of Chief State School Officers (CCSSO). 2010. *Reaching Higher: The Common Core State Standards Validation Committee—A Report from the National Governors Association Center for Best Practices and the Council of Chief State School Officers*. Washington, DC: NGA Center and CCSSO.

Comparing Ratios



Mathematical Focus

- (6.RP.3) Use ratio reasoning to solve real-world and mathematical problems.

Potential Challenges and Misconceptions

Too often students apply algorithms or formulas erroneously. Helping students develop the ability to estimate and compare ratios informally before introducing such techniques provides opportunities for students to reason quantitatively while developing conceptual foundations for later work.

In the Classroom

Present the following information to students:

> There were 20 problems on the quiz.
>
> Student A answered 4 problems correctly for every 1 problem answered incorrectly.
>
> Student B answered 7 problems correctly for every 3 problems answered incorrectly.

Have the students work individually for about four minutes, writing down everything this information tells them. Circulate with a clipboard as they write, noting those students who have several ideas and those that have fewer. Then have students turn to their partners to exchange ideas. Again circulate, this time paying attention to the words the students use to describe and compare the ratios.

Have pairs share one idea at a time with the whole group for as many times as it is possible to do so without repeating. Record each of the comments for all to see. With each suggestion, ask other students if they agree or disagree and discuss as necessary. Consider asking the following questions if no one brings up these ideas:

- *How many problems on the quiz did Student A solve correctly? How do you know?*
- *Who solved more problems correctly on the quiz, Student A or Student B? How do you know? Does anyone else have another way to find this answer?*

Next display the questions below and encourage students to share their thinking with the class.

> Which of the following ratios would you rather have describe how your correct answers compared to your incorrect answers?

5:6 or 6:5
10:3 or 30:9
7:3 or 14:6

Assign the *Which Ratio Do You Want?* reproducible on page A8 in the appendix for more practice.

Meeting Individual Needs

Encourage some students to create tables, tape diagrams, or double number lines of equivalent ratios to help them compare ratios.

REFERENCE/FURTHER READING

Sharp, Janet M., and Barbara Adams. 2003. "Using a Pattern Table to Solve Contextualized Proportion Problems." *Mathematics Teaching in the Middle School* 8 (8): 432–39.

WHICH RATIO DO YOU WANT?

Name: Date:

For each situation, look at each ratio and quickly decide which ratio you would choose. Then explain how you might convince others about your decision. Change your decision and argument if you find it is necessary.

1. You have just won a prize. You will be paid the money in five-dollar and one-dollar bills. You want the biggest prize you can get. Which ratio of five-dollar to one-dollar bills should you choose, 3:4 or 7:5? Justify your thinking.

2. You and a friend have been assigned to wash dishes after the school party. You do not like to wash dishes. Would you prefer to wash 7 dishes for every 11 your friend washes or 14 dishes for every 19 your friend washes? Justify your thinking.

3. Your gym teacher has assigned everyone to do sit-ups and jumping jacks but will let you decide how many sit-ups you do for each jumping jack. You prefer jumping jacks. Which ratio of sit-ups to jumping jacks will you choose, 5:6 or 11:12? Justify your thinking.

How Big Are These Chairs?

A36–A37

Mathematical Focus

- (7.RP.2) Recognize and represent proportional relationships between quantities.
- (7.RP.2a) Decide whether two quantities are in a proportional relationship.
- (7.G.A.1) Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Potential Challenges and Misconceptions

Within textbook problems, students' exposure is often limited to "nice" numbers with ratios or scale factors provided, which is not the case when applying mathematical ideas to real-world data. When confronted with the latter, many students do not know how to proceed. Making comparisons between a typical chair and scaled versions of that chair is a strategic way for students to develop an understanding of how an object can be dilated—that is, enlarged or reduced—by given scale factors, and how to determine such data in real-world settings.

In the Classroom

One teacher provides her students with pictures of a child, Emma, sitting on three different-size chairs, all of which are proportional; the ratio of the large to the middle chair is the same as that of the middle chair to the small chair. (See *Giant and Tiny Chairs* on pages A36–A37.) Before she challenges them to use the pictures and Emma's measurements to discover how tall each of the three chairs is, she goes over the measurements that are given to ensure the students understand what a sit measurement is, where the top of Emma's boots are, and what constitutes the height of the chairs. This teacher suggests the students work in pairs and agree on how to begin. She also suggests that each student take his or her own measurements (using a standard ruler) as a means of double-checking the work.

As she walks around the class, this teacher notices some students really struggling to determine how to go about solving the problem. Rather than let them struggle to the point of frustration or quitting, she brings the class back together, even though the students are not finished answering all the questions. She suggests that it might be helpful if some student volunteers share their *strategies* (not answers). She does this often to ensure that even students who may struggle have an opportunity to choose among strategies others are using and that are effective for solving these problems. She begins this sharing by asking Marissa and her partner how they worked on the problem. Marissa explains, "I started by finding the length and width of the medium-size chair and multiplied that length and width to find the surface area of the seat." As soon as Marissa makes that statement, many of the other students agree that they did the same thing.

Next, the teacher asks Nicole and Tyler to share what they've done. These partners project a table they have made to organize their work. Nicole explains that she kept getting confused by the ratios and the scale factors, so they made the table to help them organize their data.

Tucker then volunteers to explain how he is determining the heights of the chairs. He reports, "I measured the biggest chair by measuring from the top of the chair to the seat, and I will compare it to Emma's height."

This teacher then encourages the students to choose one of the shared strategies if they've been struggling or to finish their calculations if they already have a method that is working for them.

Meeting Individual Needs

For students who need more support, it may be helpful to enlarge the pictures and break the problem down into different components. Have them do all the measuring and record the measurements in a table. After that is finished, suggest they find the ratio between the normal-size chair and the largest chair, followed by the ratio between the normal-size chair and the smallest chair. Keep in mind that if the students begin with the smallest chair and work their way to the largest chair, the scale factors will be whole numbers, but if they work their way from the largest to the smallest, the scale factors will be fractions.

REFERENCE/FURTHER READING

Van de Walle, John, Jennifer Bay-Williams, LouAnn Lovin, and Karen Karp. 2013. *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6–8*. 2d ed. New York: Pearson.

GIANT AND TINY CHAIRS

Name: Date:

Emma sat on three different-size chairs (see pictures). The seat of each chair is a square. Following are Emma's measurements:

- From the seat to the top of her head: 54 cm
- From her back to the top of her boots: 47.5 cm
- The height of her boots: 17 cm
- The length of her back to her knee: 25 cm
- The length of the smallest seat is $\frac{1}{4}$ the length of the largest seat.
- The smallest seat measures 9 inches on each side.



1. Based upon her sit measurements, determine the approximate heights of each chair and the dimensions of each seat.

2. If we stacked the smallest chairs one on top of another, how many chairs would we need to stack to reach the same height as the largest chair?

3. How many of the smallest seats would fit onto the largest seat?

4. What scale factor was applied to the normal-size chair to get the height of the smallest chair?

5. What scale factor was applied to the normal-size chair to get the height of the largest chair?

6. If you applied a scale factor of $\frac{5}{2}$ to the normal-size chair, what would the new height be?

7. If you applied a scale factor of 8 to the smallest chair, what would the dimensions of the new seat be?

Taxes, Tips, and Commissions

A47, A52–A53

Mathematical Focus

- (7.RP.3) Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Potential Challenges and Misconceptions

Determining the tax for an item and figuring the amount of tip to leave for the wait staff are everyday life skills that many students are beginning to realize they need. Students who lack flexible thinking with percents will not be able to compute tips quickly and may overtip or undertip. Accuracy is also important when determining such things as taxes and commissions. This financial literacy will affect your students throughout their lives, so it is important that they develop both precision and flexibility.

In the Classroom

To begin his lesson, one teacher asks his students to use their individual whiteboards to indicate how much money they would leave for a tip if their meal cost thirty dollars. After they record their answers, he tells them to hold up their whiteboards, and he writes the students' responses on the class board. The values vary from one dollar to ten dollars. The teacher asks his students to share their strategies for determining the tip.

Avery offers, "My dad always leaves five dollars, so that is what I think the tip is."

Josh shares, "My mom is a waitress, so we always multiply the total bill by two and move the decimal point one place to the left, so I would leave six dollars."

Jackson adds, "My sister has a card she uses, and she shows the thirty dollars, then decides what percent tip to leave and puts down the amount on the card."

Simon nods his head and adds, "My dad rounds up the amount [of the meal], then divides by three, so he would leave a ten-dollar tip."

After his students report out, this teacher asks them to think individually about why people leave tips before telling the students to turn to their shoulder partners and share ideas.

After an allotted period of time, he asks student volunteers to report their findings. Some students say people leave tips to show they appreciate the service, others say it's because wait staff do not make enough money, and still others say it's because it is expected of them. The teacher discusses the custom of tipping that exists in the United States before asking his students what they know about sales taxes. This teacher also tells them that the customary tip is between 15% and 20%, depending on how good the service is.

He presents the following problem, again having students think individually, share with their partners, and then report their ideas.

> Diego bought a soccer ball that cost $15. The sales tax is 4% in his state. How much was the tax on the soccer ball? ($0.60) How much did he have to pay for the soccer ball? ($15 + $0.60 = $15.60)

Next, he introduces the concept of commissions by explaining that as he was working his way through school, he sold hot dogs at Fenway Park. His salary was only $2.50 per hour, but his commission was 45 percent on hot dogs, which he sold for $1.25 each. His typical workday was four hours, and he sold an average of 235 hot dogs per game. The teacher challenges the students to calculate his commission as well as how much money he made during each ball game (commission: $132.19; salary: $10; total pay: $142.19). He continues his story by telling them he worked eighty-one days per season and asks them how much money he made on average from opening day through the final game of the season (commission for season: $10,707.39; salary per season: $810.00; total pay per season: $11,517.39).

Next he assigns the *Match It and Prove It* activity on pages A52–A53 in the appendix. After students complete the matching activity, you can have them meet in small groups to discuss their work. Ask them to identify problems they found more and less challenging and why they think this was the case. Have them share the techniques they used to prove their choices were correct. Encourage students to note the similarities and differences among their strategies. If no one in the group has used an equation, challenge them to do so.

Meeting Individual Needs

The matching activity can help to relieve anxiety in some students because they know all of the correct answers are given. You may want to make copies of the *Percent Templates* reproducible from page A47 in the appendix available for students who would benefit from this organizational structure.

REFERENCE/FURTHER READINGS

Ercole, Leslie K., Marny Franz, and George Ashline. 2011. "Multiple Ways to Solve Proportions." *Mathematics Teaching in the Middle School* 16 (8): 482–90.

PERCENT TEMPLATES

Name: Date:

Value	Percent

Value	Percent

Value	Percent

Value	Percent

MATCH IT AND PROVE IT

Copy the following problems and answers onto card stock and cut them out. Working in pairs, students shuffle the question cards and spread them faceup on their tables or desks. Next, they shuffle the answer cards and place them facedown in a pile. The first student turns over an answer card, and both students compete to match a problem to the correct answer, and prove that the match is mathematically correct. The person who matches the question and answer correctly keeps both cards. If neither student gets the correct answer, the answer card is placed at the bottom of the answer card pile. Students take turns turning over the answer card. (*Note:* These cards can also be used in *Play It Forward*, as there are multiple question types included in this set.)

Questions

A A tip of 15% for a bill of $45.	B A 6% tax on a video game that costs $19.99.	C The cost of a $15 book after a 7% sales tax.
D A 20% tip for a meal that cost $32.	E School supplies cost $72. A tax of 5% is added to the subtotal. What is the total bill?	F Jackson pays $156 for his sports equipment. The tax is 5%. What is the final cost?
G Isabelle gave a $4.50 tip for a meal that cost $30.00. What percent tip did she give?	H The sales tax is 4.5%. If you buy a bicycle that costs $150, how much will you pay?	I The sales tax in one state is 4.5%. How much tax will you pay on a skate board that costs $78?
J Mrs. Gates left a $16 tip for a meal that cost $80. What percent tip did she leave?	K If you paid $50, including tip, for a meal that cost $42, what percent tip did you leave?	L Josie bought a friend a birthday present that cost $38 plus 5% tax. How much money did she spend?

M	N	O
Matthew paid $84 on a bill that came to $63. What percent tip did he leave?	Julia's mom sells real estate. She sold a house for $189,000 and received a 3% commission. How much money did she earn in commission?	Noah has a paper route and receives a 2% commission on the papers he sells. If he sells $145 worth of papers, what will his commission be?

Answers

$6.75	$1.20	$16.05
$6.40	$75.60	$163.80
15%	$156.75	$3.51
20%	19%	$39.90
$33\frac{1}{3}$%	$5,670	$2.90