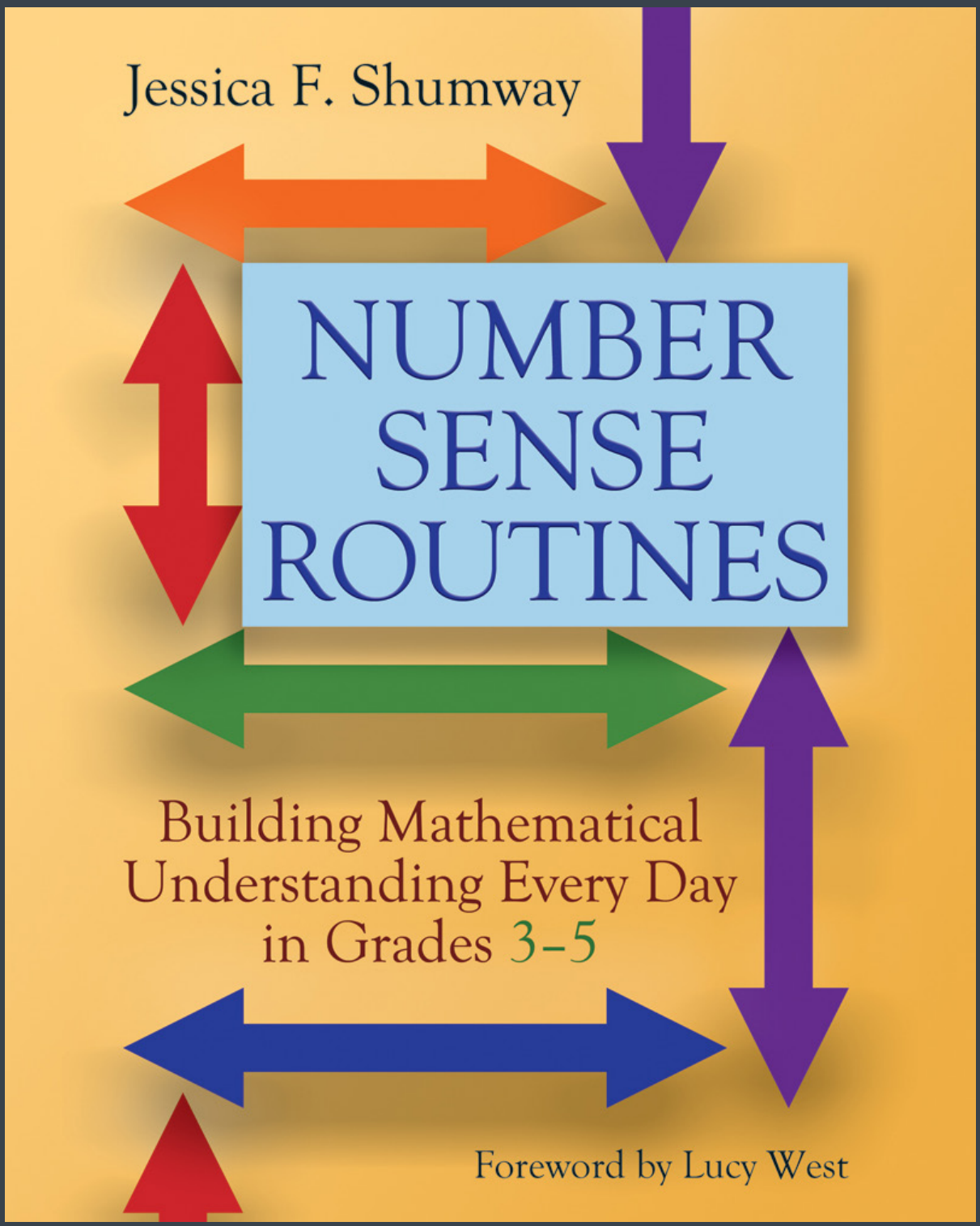
Study Guide



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Summary

Get ready to immerse yourself in number sense routines and to be challenged to reignite the way in which you envision the development of number sense in your students. The ideas in Jessica Shumway's book take on life as you enter the classrooms of teachers who are using a myriad of routines in ways, which are not "routinized." This book invites you to sit beside the author, try out math tasks, listen to children and teachers, and consider strategies for planning and facilitating your own number sense routines. This is not just another book with a list of activities. Jessica journeys with us as we learn how to respond to our students' needs with purposeful and intentional planning of number sense routines.

The first section of the book, *Building Number Sense Through Routines,* consists of two chapters. In this part of the book, Jessica clearly lays out the importance of number sense development and offers us a rationale for the use of number sense routines. With accessible and lively writing, we are asked to consider the importance of students "building understanding from within and taking an active part in constructing their number sense" (11). A clear and handy table of routines and questioning strategies provides a reference for you as you continue your journey through the book.

The second section of the book is a treasure box of chapters, which introduces number sense routines such as Counting Routines and Quick Images. These routines provide high leverage opportunities that target the number sense ideas that third, fourth and fifth grade students are developing. The visual images of student work, and the "What's the Math" boxes, provide you with support as you analyze the mathematics underlying the routines. The student cases and classroom episodes give us a glimpse into the practical applications of the routines as well as uncover the nuances that teachers need to pay attention to. There are a variety of suggested resources, questioning strategies and supports for getting started and making number choices.

The third section of the book, *More Than Just the Routine,* invites you to consider the role of classroom community. In this section you will explore the importance of math talk, mistakes as learning opportunities, and the process of reflection. You will be asked to join Jessica as she connects the number sense routines to the building of a strong and supportive community of learners. In the final chapter, *Planning Responsive Number Sense Routines,* there is an opportunity to dig into the challenging work of being responsive to individual student needs and how to plan for the whole class as well.

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This guide is structured for a study group. While there are many opportunities for collegial conversations, individuals will also find the study guide to be a useful tool for reflection. The following questions and activities will help readers to extend the ideas of the book and to apply them to their own classrooms. You will find questions that will pull you back into thinking further about a teacher's moves or the ideas of specific students. There are quotes to consider that will stimulate conversations around beliefs and practices, and there are suggestions for classroom applications.

So grab a highlighter, plenty of sticky notes, and a few colleagues! Then get ready to join Jessica and a team of teachers and students as they immerse you in gathering a wealth of ideas for number sense routines, tools for planning and instruction and effective strategies for supporting your students' progress toward numerical literacy.

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Introduction

Discussion (15 minutes)

- At the beginning of the introduction, there are two quotes that capture the challenges teachers face as they address the number sense gaps that they recognize as they work with upper grade elementary students. Share some examples of your own experiences and challenges in supporting students who have difficulty with grade level concepts.
- The author mentions how upper elementary students are working toward ideas that move them beyond whole numbers to fractions, and developing generalizations about place value. Work with your study group to identify two or three ideas related to fractions and place value that you focus on at your grade level.

Quotes Worth Discussing (10 minutes)

- Discuss this statement on page 2: "*This book is about tapping into every child's innate sense of number."* This statement sets the premise that all students have an innate sense of number. What reactions do you and your colleagues have toward this statement?

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Chapter 1
Number Sense: What Does It Mean?

Let's Do Some Math! (10 minutes)

- Solve this problem on page 7 in two different ways.

  On our field trip to the Great Salt Lake, we collected brine shrimp. There are 9 brine shrimp in each test tube. If there are 12 test tubes, how many brine shrimp were collected from the Great Salt Lake?

- Compare your methods with a partner. What ideas about numbers and number relationships did you call upon?

Discussion/Sharing (15 Minutes)

- Discuss the work of the fourth-grade students in Figure 1.1 on page 8. What does each student seem to understand? What ideas may each student still be developing? Connect their ideas to the strategies you used in "Let's Do Some Math!"
- Return to the work of the students in Figure 1.1 and the teacher's description of their understandings. Connect the list of number sense understandings and skills on pages 11 through 13 to the evidence of student understanding.
- Examine the choices the teacher makes during the discussion in terms of the partnerships she establishes for a pair-share. Describe the rationale she uses and the understandings that individual students brought to the problem.

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes and the impact on your understanding of number sense.

- *"As teachers, having a strong understanding of what number sense is and all its components and complexities will improve our ability to plan experiences that will develop and nurture our students' number sense" (10).*
- *"As students build their number sense, mathematics takes on greater meaning" (11).*
- *"By the time they come to school, children already have a store of informal math knowledge" (13)*

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Putting Ideas into Practice (15 Minutes)

- Revisit the list of number sense understandings and skills on pages 11 through 13. Think of a student you would describe as having number sense. What evidence do you have that the student has many of the components of number sense? How might you assess the student to get more evidence?
- An important part of building number sense is knowing that there are multiple ways to solve problems. How do you support the development and sharing of multiple solution paths? Examine an upcoming lesson or task or routine with which you will engage your students. How can you support students in encountering multiple ways to solve the problem?

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Chapter 2
Improving Number Sense: Routines That Are Not Routinized

Let's Do Some Math! (10 minutes)

- Start at 0 and count by sixths to 2. What patterns do you notice? What are some fractions you landed on as you counted by sixths that could be renamed with an equivalent fraction?
- Start at 0 and count by twelfths to 2. What patterns do you notice? Can you use those patterns to predict how many times we would have to count by twelfths to get to 10?
- Compare the counting by sixths to the counting by twelfths. What observations can you make? Share those observations with your study group!

Discussion/Sharing (15 Minutes)

- Choose one of the fifth-grade students featured on pages 16 through 20. What mathematical idea(s) is the student working with (you can refer back to pages 8 through 10)? How does the Counting Around the Circle routine mentioned in the vignette support the students in developing number sense?
- Follow the learning path of one of these students (21-25): Brooke, Leila or Martin. How do the experiences with the number sense routines impact the learning path? Give specific examples.

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes.

- *"These number sense routines are not 'autopilot' activities, but opportunities for meaningful, connected mathematical understanding" (16).*
- *"The number sense routines explored in this book are 'responsive' routines- they are responsive to students' discussions, understandings, and learning needs (19).*
- *"Routines provide a comfortable predictability, but at the same time, we plan routines that will keep students challenged, provide opportunities to practice using their numbers sense, and reteach when necessary" (19).*

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Putting Ideas into Practice (15 Minutes)

- What routines do you already have in place that support a classroom culture of inquiry and risk taking? How will the ideas in this chapter help you refine the routines you may already have in place? Examine your math block and develop a plan for including the routines.
- On pages 26 and 27, there is a table of number sense routines which are responsive and support students' discussions, understandings, and learning needs. Choose a set of ideas from the table and discuss how the idea will support your mathematical goals for your students.

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Chapter 3
Visual Routines: Linking Visual and Symbolic Understandings of Quantities

Let's Do Some Math! (10 minutes)

- Examine the chart on page 52. Create a visual image for each of the three problem types. Connect the word problem and the possible equations to each of your images. What do you notice? How do your visual images assist you in seeing the structure of each problem?

Discussion/Sharing (15 Minutes)

- Discuss the differences and similarities between subitzing and spatial structuring. Refer to Box 3.1 and Box 3.2 (33).
- On pages 35 through 39, follow the teacher's moves as she introduces the Quick Images routine to a group of third graders. How does she facilitate connections among students' ideas and support classroom community? How does she make explicit the mathematics of the routine?
- A group of fifth grade students (42) is engaged in a discussion about arrays (of muffins!). What evidence do we see of students coordinating two composite units and operating on them? Note the description of the progression to unitizing described in the first paragraph of page 42.
- The vignette on pages 47 through 50 describes Jessica's work with a third-grade class who was examining arrays. When shown the figure on page 47, the responses of 15 and 18 were offered. Discuss with your study group the challenges and benefits of the array model. What experiences do students need to have in order to makes sense of this structure? How did the students in this class progress with ideas as they moved to the array on page 48?
- On page 50, this statement is made: "*Moving from additive to multiplicative reasoning is one of the most difficult leaps in mathematical thinking."* Discuss how area/array problem types work along with visual images support this leap.

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes. Reflect on the routines in this chapter and connect them to the quotes.

- *"Subitizing and spatial structuring are important to our students' abilities to see*

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and conceptualize quantities" (33).

- *"By the upper elementary grades, students need to be able to use this concept of unitizing and use composite units in repeated addition and subtraction" (42).*
- *"Numbers do not mean much to children if they do not have the visual images to with the number words or written numerals" (53).*

Putting Ideas into Practice (15 Minutes)

- Choose a student to interview. Begin with a composite units image, then move to an array image. Pose questions such as the ones you read in this chapter to see what the student notices about each image. Does the student count by ones? Skip count? Repeatedly add? How does the student represent the image with an equation?
- Spatial structuring is a main theme in this chapter. Analyze the visual routine, Quick Images, in this chapter and discuss how you see spatial structuring in each of variations. How can you use this routine to bolster understandings of multiplication and division? Of fractions?

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Chapter 4
Counting Routines: Understanding the Number System and Number Relationships

Let's Do Some Math! (10 minutes)

- Examine figure 4.1 on page 56. Share two ways you would justify the number that belongs in the box.
- Solve this problem in two ways. Share your strategies with a partner and compare your methods.

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Discussion/Sharing (15 Minutes)

- Examine Figures 4.6 and 4.7 on pages 61 and 62. Read the discussion among the students and follow the line of thinking in the written work on the charts. How are the students making use of place value understandings? How are they using decomposing strategies and observations about patterns?
- We can support our students' number sense by being purposeful about how we represent visual representations of counting sequences. Choose one of the Counting Sequences described on pages 65 through 67. Work with your study group to design another format for recording the count. Discuss what patterns and relationships could become teaching points.
- Analyze the vignette of the fourth graders and their teacher Jalyn (68-75). Consider the following questions:
    - How does Jalyn use estimation strategies to engage her students in noticing and explaining patterns? Find specific examples of students adjusting or justifying their estimates.
    - How does Jalyn hold students accountable for explaining and justifying their observations? Find specific examples.
    - Although the class is counting by a small number (4), the discussion moves into larger number patterns and multiplication with larger numbers. How does Jalyn press the students to consider relationships with larger numbers?

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Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes. Relate your discussion to the goals you have for your students' understanding of the base ten number system.

- *"Counting is not just a memorized sequence" (56).*
- *"Visual representations in the form of number grids and number lines provide students with opportunities to think about and visually understand numbers, their relationships, and their place in the number system" (64)*
- *"Estimation is an important skill for computation" (69).*
- *"Number sense routines are a form of formative assessment because we can really delve into the whys and hows of mathematical concepts" (74).*

Putting Ideas into Practice (15 Minutes)

- Use current information you have about the needs of your class in terms of developing more efficient strategies for adding numbers (the range of numbers according to your grade level). Develop a three-day sequence of counting routines based on those observations. Refer to Boxes 4.1 and 4.2 (pages 64 and 68) for ideas to get you started. Keep notes on how the students responded to the routines and how you made adjustments when needed. Share your learning with a partner.
- On pages 75 through 79, we saw examples of students using counting strategies and visual representations to make sense of a word problem. Choose a word problem from your curriculum or create a word problem for your students to solve. Encourage them to show their strategy with a visual representation. What do you notice about their strategies and their representations? How are they applying their number sense in problem solving situations? What counting routines might support the students' number sense?

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Chapter 5

Playing with Quantities: Developing, Representing, and Generalizing Number System Understandings

Let's Do Some Math! (10 minutes)

- Ways to Make 148: work independently and then with a partner to represent 148. Compare your ways of thinking about 148. Then create expressions with this constraint: you must use multiplication and division and include at least one fraction or mixed number in your expression.

Discussion/Sharing (15 Minutes)

- Discuss the differences and similarities between the two routines "Today's Number" and "Ways to Make a Number." In what ways do these routines support an understanding of place value ideas and the operations?
- Examine the student work on pages 84 through 85. Return to the big ideas listed on page 82. What evidence do you see of students making use of or trying out some of those ideas?
- The vignette on pages 86 through 93 features fifth graders working with Ways to Make a Number with two tenths. How does the choice of this number (0.20) provide students with an entry point into thinking about
    - fractions as division
    - the commutative property
    - equivalent fractions
    - grid models
    - number lines

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes. Reflect on the routines in this chapter and connect them to the quotes.

- *"Flexibility with numbers is the key to invented strategies and effective methods for solving problems and operating on numbers" (81).*
- *"When students understand the ways number work in our number system, the way they look at and use numbers changes" (82).*

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Putting Ideas into Practice (15 Minutes)

- On page 83, the author writes: "Sometimes the difficult part of this routine is knowing what to look for in students' Ways to Make a Number representations and knowing how to highlight important math concepts during the discussion of the routine." Engage your class in a Ways to Make a Number routine. Collect their work and discuss (with your study group) decisions you would make in using the variations of students' responses.
    - Think about what mathematical concepts could be highlighted using specific examples.
    - What connections could be made across equations or representations?
    - How could you press students to use a pattern to create additional equations?
- Plan a Today's Number routine for a small group of students. Use the questions and statements in Box 5.2 on page 93 to get you started. Think carefully about the number you choose and the ways you will ask students to interact with the number. What do you notice about how the students explore the number? What additional questions might you ask?

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Chapter 6

Learning From Each Other: Building a Strong Community of Learners Through Math Talk, Mistakes, and Reflections

Let's Do Some Math! (10 minutes)

- Choose a routine from one of the previous chapters. Take turns doing a "rehearsal" with your team. Use the Math Talk Tips on pages 160 and 161 to plan the questions you will ask. Reflect as a team on how the conversation supported the number sense goals.

Discussion/Sharing (15 Minutes)

- On page 102 the author lists four skills that are critical in developing a community in which mathematical discourse is central. Discuss how these four skills have been present in the vignettes that you have read in previous chapters. Then examine the classroom episode "Talking About Visual Quantities" on page 99. How does the teacher support mathematical discourse? What evidence do you see in the students' responses that show the impact of the teacher's purposeful support?
- Analyze the vignette on pages 112 and 113. How are the students using reflection to support their learning and the learning of others?

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes. Reflect on the steps you have taken to develop a classroom community that support sense making.

- *"To successfully share our thinking with each other and learn from one another, we must build a strong and supportive community of learners" (97).*
- *"Discussion among students is an absolutely critical component in their mathematics development" (99).*
- *"Our language is powerful and sets a tone in the classroom. The language we use can build the idea that mistakes are opportunities for learning and that learning is a process" (111).*

Putting Ideas into Practice (15 Minutes)

- Choose one area of Math Talk that you wish to focus on. Prepare a routine and focus on intentional teacher moves to support your focus. Record one of your

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Classroom Routine sessions and analyze how you supported your goal for supporting mathematical discourse. How did your students respond? What would you like to improve upon? Try a second routine and repeat the process. What do you notice? What are areas you would like to focus on next?

- On pages 109 through 112, mistakes are discussed as opportunities for learning. Choose a routine with which to engage your class, and plan a three-day sequence. Anticipate misconceptions or mistakes that might surface, and prepare for them using the phrases and questions on page 111. Over the course of the three days, keep track of mistakes that students made and how you responded to them. Share your reflections and next steps with a partner.
- Reflection is an important component of community building. Both individual and group reflections allow the classroom community to grow and support its members. Over the next few weeks, include reflection as part of your mathematics discussions. Use the questions and prompts on page 113 as a guide. Share the reflections and what you learned about your students with your study group.

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Chapter 7
Planning Responsive Number Sense Routines

Let's Do Some Math! (10 minutes)

- Choose a sequence of upcoming mathematics tasks or lessons. Do the math of the lessons yourself and anticipate your students' responses. How would these lessons benefit from additional work with routines?

Discussion/Sharing (15 Minutes)

- Jeff, the fourth-grade teacher, plans to support conceptual understanding of multiplication by including several routines (115). Discuss how he
    - uses quick images to support his goals
    - connects the routines to word problems
    - intentionally chooses numbers
- Analyze the "next steps" that each teacher plans in response to their students' current understanding. Vignettes begin on page 121.

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes. Reflect on your own practice and identify a goal to work on based on the ideas in these quotes.

- *"Students construct learning based on prior learning; therefore, our instructional starting point should be what students know and can do" (116-117).*
- *"Assessment and instruction are symbiotic" (119).*

Putting Ideas into Practice (15 Minutes)

- Choose three pieces of student work that you have collected related to routine you have been implementing. Discuss your analysis with your study group. What commonalities or trends do you see in the students' responses? What different needs are evidenced in the work samples?
    - Identify a small group of students who have similar needs. Plan a five-day sequence of routines to address their needs. Use formative assessment strategies to monitor their progress. At the end of the five days, reassess and examine their current understanding.

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Conclusion
A Place to Begin

Discussion/Sharing (15 Minutes)

- On page 126, the author encourages us to "start small." Take time with your study group to reflect on the small steps, challenges and accomplishments that have been part of your journey to implement Number Sense Routines.
- There are five ideas for starting out on page 127. Discuss which of those ideas you feel would best meet your own goals for making number sense routines successful in your classroom.

Quotes Worth Discussing (15 Minutes) Discuss your interpretation of these quotes.

- *"As students develop their number sense, their overall mathematical understanding improves" (126).*
- *"Developing one's number sense is never 'done;' therefore, enjoy the voyage of the continuous development . . ." (128).*

Putting Ideas into Practice (15 Minutes)

- This last chapter begins with quotes from two different students who are sharing how they feel about the impact of routines on their number sense. Choose a few students in your class to interview or prepare a survey so that you can gather some comments from your students. Reflect on how your students
    - feel about mathematics; what is their disposition
    - connect the work with routines to their own ability to solve word problems
    - think about efficient strategies and the use of multiple strategies
    - see the role of visual images (IE number lines, arrays, charts)
    - view the role of mathematical discourse
- Look back over the work you have done in this study group. What changes do you see in your own understanding of the term "Number Sense"? What impact has this work had on your students?

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- Make a plan for how you will continue to share and build on your study group's ideas! Keep the collaboration going.

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